



Mathematics: Visual Calculation Policy (May 2018)

ADDITION

Key Language: sum, total, and, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

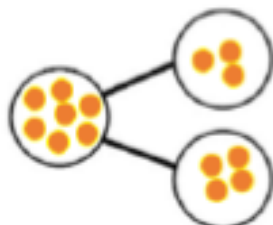
Concrete

Combining two parts to make a whole
(using a variety of resources – toys, books etc)



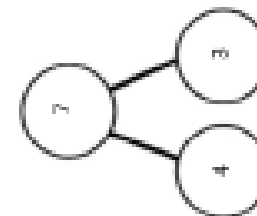
Pictorial

Part, Whole Model
(Using crosses, dots etc)

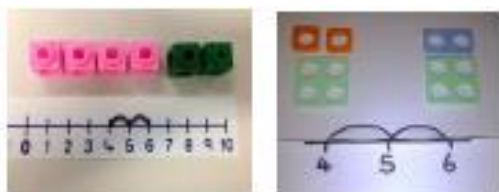


Abstract

$4 + 3 = 7$
(Four is a part, three is a part and the whole is seven)



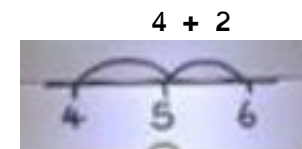
Counting on using number lines
(Using cubes or numicon)



A bar model which encourage children to count on, rather than count all.



The abstract number line:
What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2?



Regrouping to make 10 by using ten frames and counters/cubes or using numicon:

$$6 + 5$$



Children to draw the ten frame and counters/cubes



Children to develop an understanding of equality
E.G.

$$6 + \square = 11$$

$$6 + 5 = 5 + \square$$

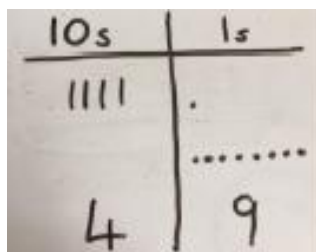
$$6 + 5 = \square + 5$$

TO + O using base 10. Continue to develop understanding of partitioning and place value

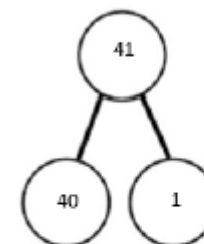
$$41 + 8$$



Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones.

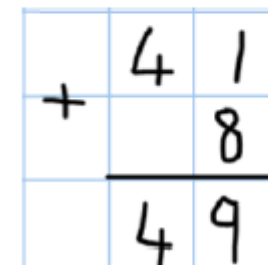


$$41 + 8$$

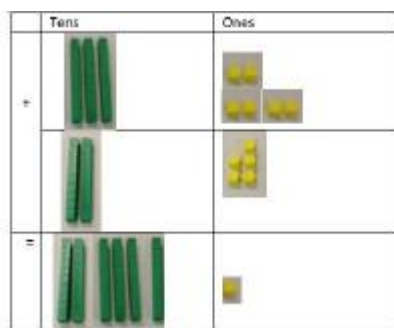


$$1 + 8 = 9$$

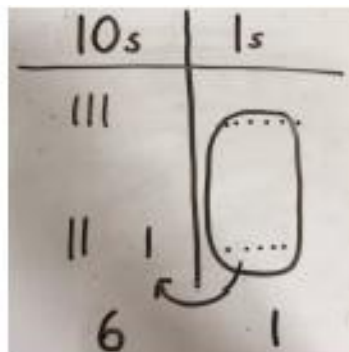
$$40 + 9 = 49$$



TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. $36 + 25$



This could be done one of two ways:



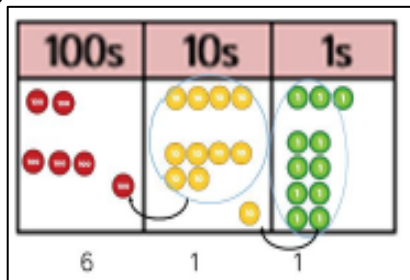
Looking for ways to make 10

$$\begin{array}{rcl}
 36 + 25 & = & 30 + 20 = 50 \\
 & & 5 + 5 = 10 \\
 & & 50 + 10 + 1 = 61
 \end{array}$$

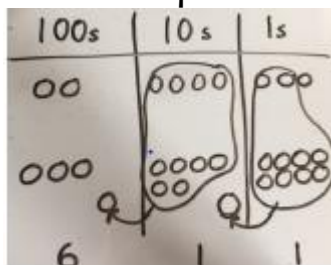
Formal method:

$$\begin{array}{r}
 36 \\
 +25 \\
 \hline
 61 \\
 \hline
 1
 \end{array}$$

Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



Children to represent the counters



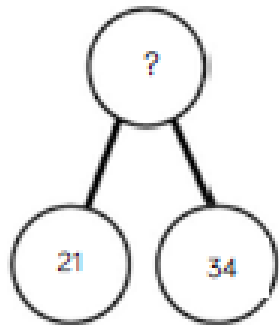
e.g.

Or draw a bar model for a word problem:

?	
243	368

$$\begin{array}{r}
 243 \\
 +368 \\
 \hline
 611 \\
 \hline
 1 \quad 1
 \end{array}$$

Conceptual variation; different ways to ask children to solve $21 + 34$



?	
21	34

Word problems:

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

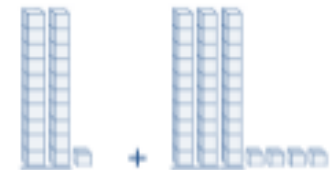
$21 + 34 = 55$. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$$21 + 34 =$$

$$\boxed{} = 21 + 34$$

What's the sum of twenty one and thirty four?



Missing digit problems:

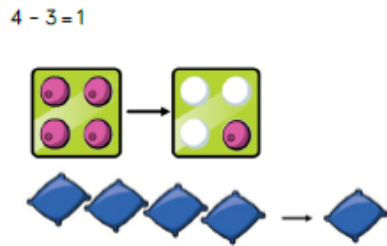
10s	1s
	?
?	5

SUBTRACTION

Key Language: take away, less than, the difference, subtract, minus, fewer, decrease.

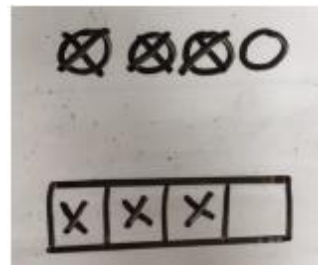
Concrete

Physically taking away and removing objects from a whole (use various objects too) rather than crossing out—children will physically remove the objects



Pictorial

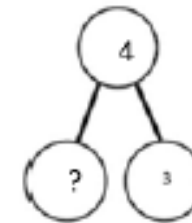
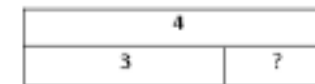
Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.



Abstract

$$4 - 3 =$$

$$\square = 4 - 3$$

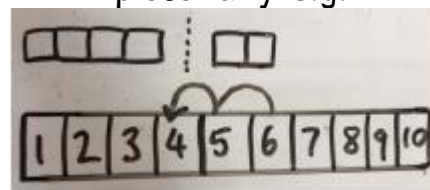


Counting back (using number lines or number tracks) children start with 6 and count back 2.

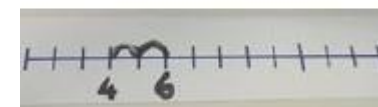
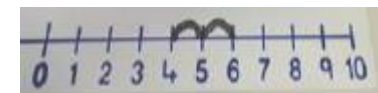
$$6 - 2 = 4$$



Children to represent what they see pictorially e.g.



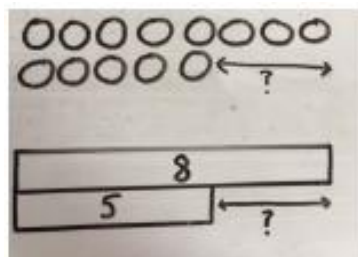
Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line



Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used)
Calculate the difference between 8 and 5



Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



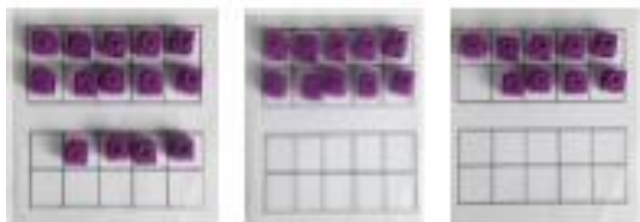
Find the difference between 8 and 5.

$8 - 5$, the difference is __?

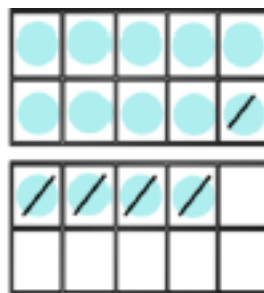
Children to explore why
 $9 - 6 = 8 - 5 = 7 - 4$
have the same difference.

Subtraction using numicon or ten frames.

$$14 - 5$$



Children to present the ten frame pictorially and discuss what they did to make 10.



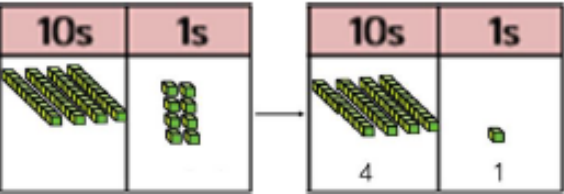
$$14 - 5 = 9$$

You also want children to see related facts e.g. $15 - 9 = 5$

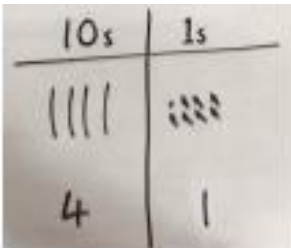
Children to represent how they have solved it e.g.

$$\begin{array}{r} 14 - 5 = 9 \\ \swarrow \quad \searrow \\ 4 \quad \quad 1 \\ 14 - 4 = 10 \\ 10 - 1 = 9 \end{array}$$

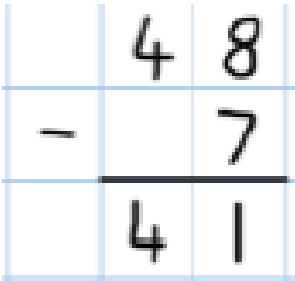
Column method using base 10.



Children to represent the base 10 pictorially.

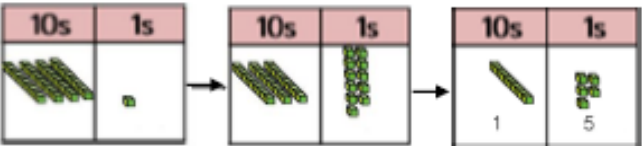


Column method or children could count back 7.



Column method using base 10 and having to exchange.

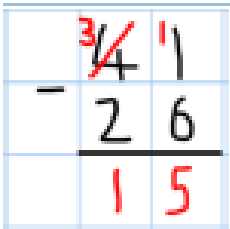
41 – 26



Represent the base 10 pictorially, remembering to show the exchange.

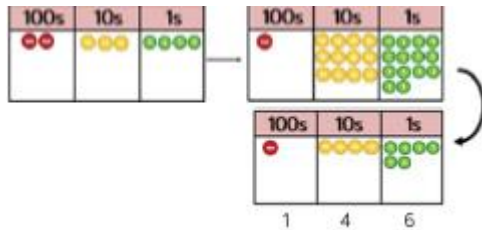


Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.

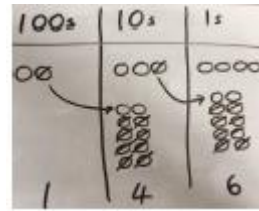


Column method using place value counters.

$$234 - 88$$



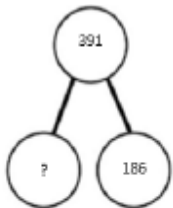
Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} \overset{2}{2}\overset{1}{3}4 \\ - 88 \\ \hline 6 \end{array}$$

Conceptual variation; different ways to ask children to solve $391 - 186$



391	
186	?

Word Problems

Raj spent £391, Timmy spent £186. How much more did Raj spend?

I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

$$\boxed{} = 391 - 186$$

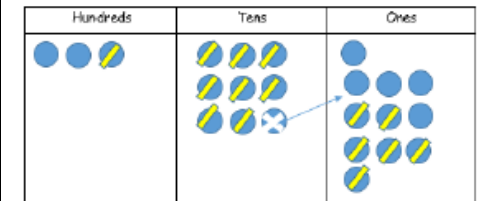
$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

Find the difference between 391 and 186

Subtract 186 from 391.

What is 186 less than 391?

Missing digit calculations



$$\begin{array}{r} 39\boxed{} \\ - \boxed{}\boxed{}6 \\ \hline \boxed{}05 \end{array}$$

What's the calculation?

What's the answer?

MULTIPLICATION

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups, 'is equal to' 'is the same as'

Concrete

Repeated grouping/repeated addition

$$3 \times 4$$

$$4 + 4 + 4$$

There are 3 equal groups, with 4 in each group.

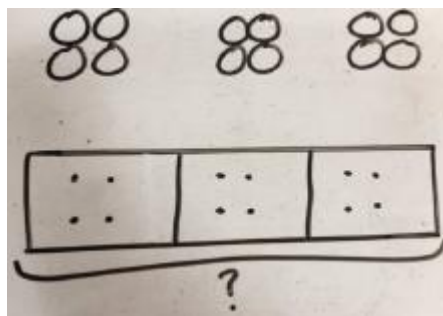


$$3 \times 4 \text{ or } 3 \text{ lots of } 4$$

(does not have to be restricted to cubes)

Pictorial

Children to represent the practical resources in a picture and use a bar model.



Abstract

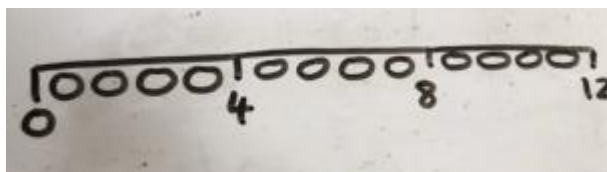
$$3 \times 4 = 12$$

$$4 + 4 + 4 = 12$$

Use number lines to show repeated groups 3 x 4

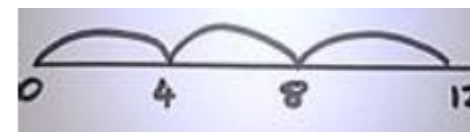


Represent this pictorially alongside a number line:



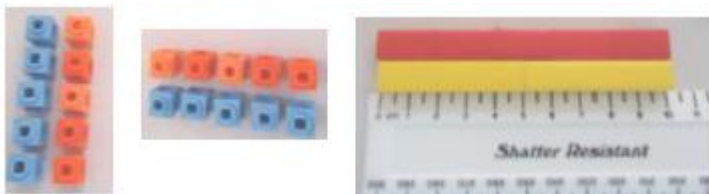
Abstract number line showing three jumps of four.

$$3 \times 4 = 12$$

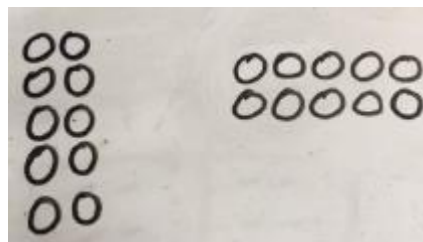


Use arrays to illustrate commutativity
counters and other objects can also be used.

$$2 \times 5 = 5 \times 2$$



Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

$$10 = 2 \times 5$$

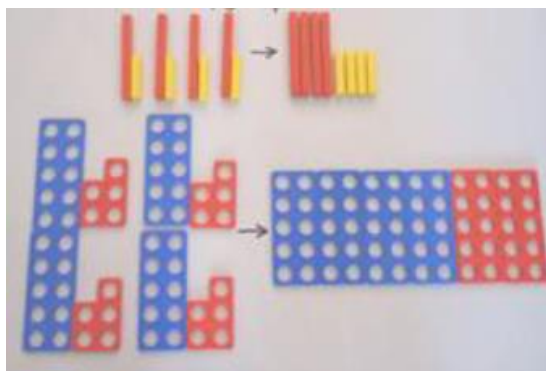
$$5 \times 2 = 10$$

$$2 + 2 + 2 + 2 + 2 = 10$$

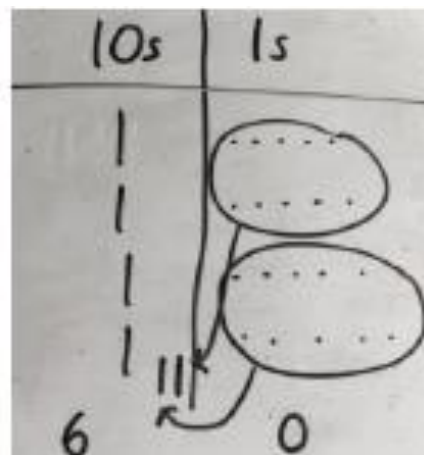
$$10 = 5 + 5$$

Partition to multiply using Numicon, base 10 or Cuisenaire rods.

$$4 \times 15$$



Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken.

$$4 \times 15$$

$$\swarrow \searrow$$

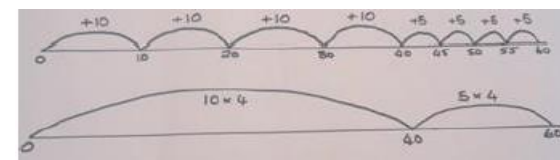
$$10 \quad 5$$

$$10 \times 4 = 40$$

$$5 \times 4 = 20$$

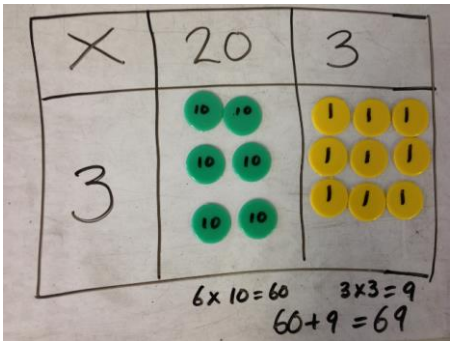
$$40 + 20 = 60$$

A number line can also be used:



Grid method using manipulatives:

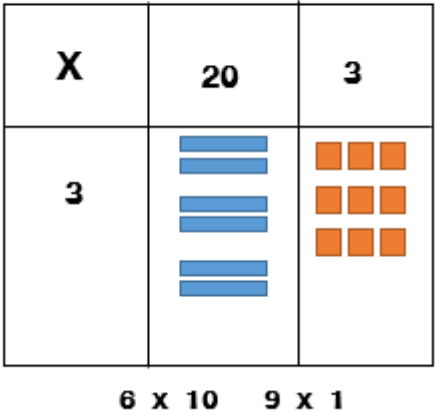
3×23



Using place value counters, cubes,
Base Ten

Drawings of manipulatives using
the grid format:

3×23



Formal written method using grids:

X	20	3
3	60	9

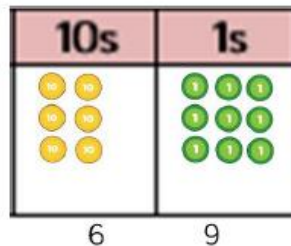
$$\begin{array}{r} 60 \\ + 9 \\ \hline 69 \end{array}$$

Moving on to 2 digits x 2 digits, 2 digits x 3
digits, 3 digits x 3 digits etc

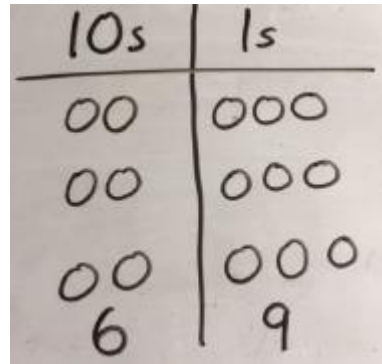
Formal column method with place value counters or base 10 (at the first stage—no exchanging)

$$3 \times 23$$

Make 23, 3 times. See how many ones, then how many tens



Children to represent the counters pictorially.



Children to record what it is they are doing to show understanding.

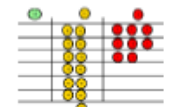
$$\begin{array}{r}
 3 \times 23 \\
 \swarrow \searrow \\
 20 \quad 3 \\
 3 \times 20 = 60 \\
 3 \times 3 = 9 \\
 60 + 9 = 69 \\
 \\
 \begin{array}{r}
 23 \\
 \times 3 \\
 \hline
 69
 \end{array}
 \end{array}$$

Formal column method with place value counters.

$$6 \times 23$$



Step 1: get 6 lots of 23



Step 2: 6×3 is 18. Can I make an exchange? Yes! Ten ones for one ten...

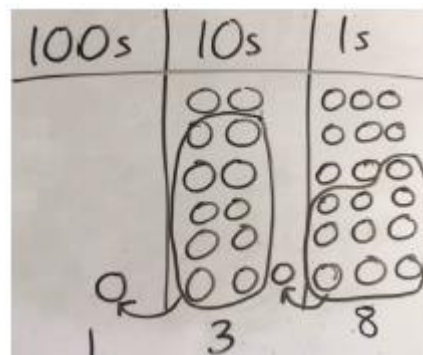


Step 3: 6×2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred...



Step 4- what do I have in each column?

Children to represent the counters/base 10, pictorially:



Formal written method

$$6 \times 23$$

$$6 \times 3 = 18$$

$$6 \times 20 = 120$$

$$120 + 18 = 138$$

The aim is to get to the formal method but the children need to understand how it works.

$$\begin{array}{r}
 6 \times 23 = \\
 23 \\
 \times 6 \\
 \hline
 138 \\
 11
 \end{array}$$

Expanded multiplication methods

Long multiplication

$$\begin{array}{r}
 286 \\
 \times 29 \\
 \hline
 54 \quad (9 \times 6 = 54) \\
 720 \quad (9 \times 80 = 720) \\
 1800 \quad (9 \times 200 = 1800) \\
 120 \quad (20 \times 6 = 120) \\
 1600 \quad (20 \times 80 = 1600) \\
 4000 \quad (20 \times 200 = 4000) \\
 \hline
 8294 \\
 1
 \end{array}$$

Short multiplication

$$\begin{array}{r}
 286 \\
 \times 29 \\
 \hline
 2574 \quad (9 \times 286 = 2574) \\
 5720 \quad (20 \times 286 = 5720) \\
 \hline
 8294 \\
 1
 \end{array}$$

When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc., they should be confident with the abstract:

To get 744 children have solved 6×124 .

To get 2480 they have solved 20×124 .

$$\begin{array}{r}
 1 \ 2 \ 4 \\
 \times \quad 2 \ 6 \\
 \hline
 7 \ 4 \ 4 \\
 2 \ 4 \ 8 \ 0 \\
 \hline
 3 \ 2 \ 2 \ 4 \\
 1 \ 1
 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6×23

23	23	23	23	23	23

?

With the counters, prove that $6 \times 23 = 138$

Why is $6 \times 23 = 23 \times 6$?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks?



Find the product of 6 and 23

$$6 \times 23 =$$

$$\square = 6 \times 23$$

$$\begin{array}{r} 6 \quad 23 \\ \times \quad 23 \\ \hline \end{array} \quad \begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$$

What is the calculation?
What is the product?

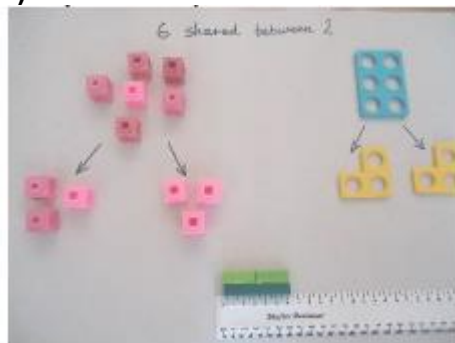
100s	10s	1s
		

Division

Key language: share, group, divide, divided by, half, 'is equal to' 'is the same as'

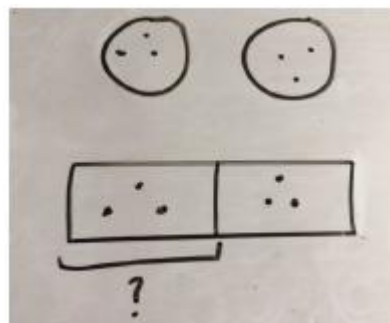
Concrete

6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates)



Pictorial

Represent the sharing pictorially.



Abstract

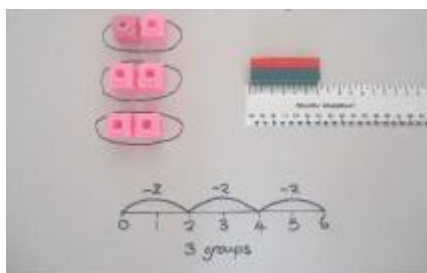
$$6 \div 2 = 3$$

What's the calculation?

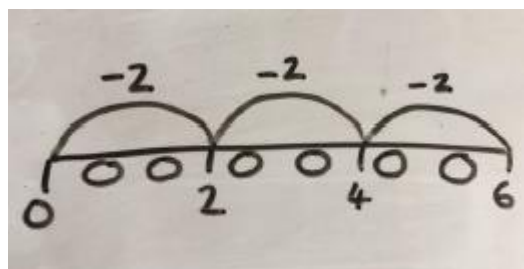


Children should also be encouraged to use their 2 times tables facts.

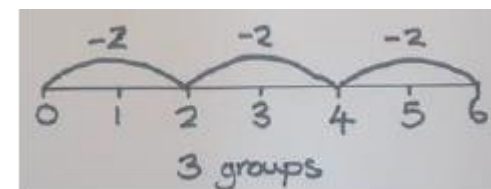
Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$



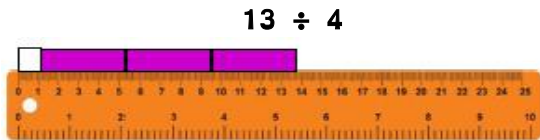
Children to represent repeated subtraction pictorially.



Abstract number line to represent the equal groups that have been subtracted.



$2d \div 1d$ with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

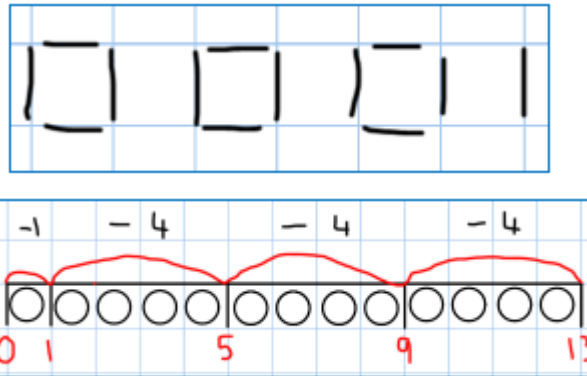


Use of lollipop sticks to form wholes—squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to have chance to represent the resources they use in a pictorial way:

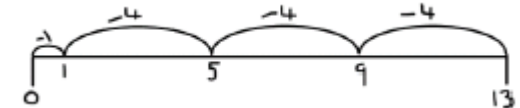


There are 3 whole squares, with 1 left over.

$13 \div 4 = 3$ remainder 1

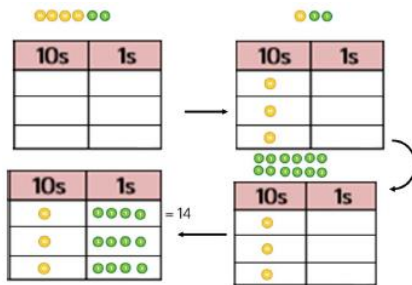
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

‘3 groups of 4, with 1 left over’

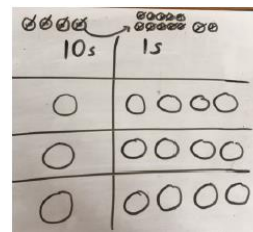


Sharing using place value counters.

$42 \div 3 = 14$



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$42 \div 3$

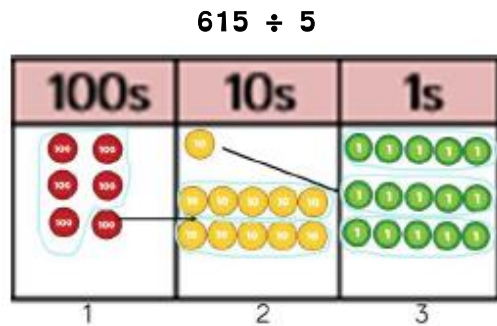
$42 = 30 + 12$

$30 \div 3 = 10$

$12 \div 3 = 4$

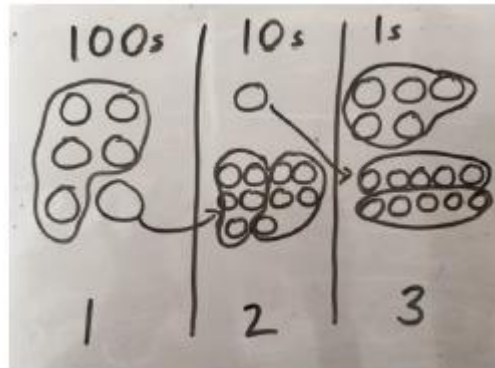
$10 + 4 = 14$

Short division using place value counters to group.



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Can be used until the children no longer needed.

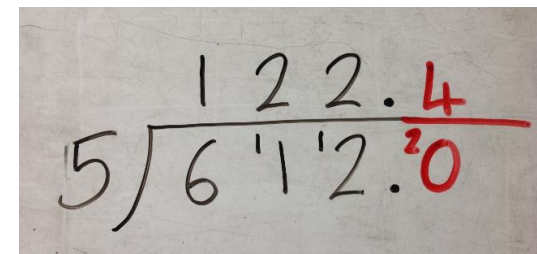
Can also be used with decimals.

Children to solve the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

Encourage the use of using times tables knowledge.

Extend onto using decimals:






Long division using place value counters




$$2544 \div 12$$

1000s	100s	10s	1s
			




We can't group 2 thousands into groups of 12 so will exchange them.

1000s	100s	10s	1s
			

We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

1000s	100s	10s	1s
			

After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

1000s	100s	10s	1s
			

After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.

Chunking method:

1	12
2	24
3	36
4	48
5	60
10	120
100	1200

$$\begin{array}{r} 2544 \div 12 \\ \underline{212} \\ 12 \overline{) 2544} \\ \underline{1200} \quad (100 \times 12) \\ 1344 \\ \underline{1200} \quad (100 \times 12) \\ 144 \\ \underline{120} \quad (10 \times 12) \\ 24 \\ \underline{24} \quad (2 \times 12) \\ 0 \end{array}$$

$$2544 \div 12 = \underline{212}$$

100 \swarrow (100×12)

100 \swarrow (100×12)

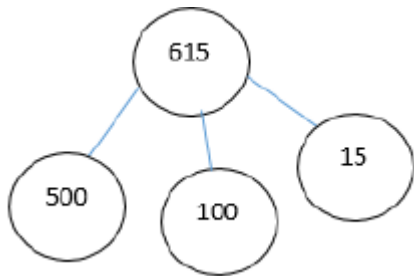
10 \swarrow (10×12)

2 \swarrow (2×12)

212

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\boxed{} = 615 \div 5$$

What is the calculation?
What is the answer?

